CS 70 SPRING 2007 — DISCUSSION #15

ALEX FABRIKANT

1. Administrivia

(1) The **last** optional "supplementary" section on Wednesdays, 4:30-7:00pm (and possibly longer than that), in 320 Soda. Bring any questions whatsoever.
(2) The last homework (homework 12) has been posted, and will be due on Tuesday, 05/08.
(3) We'll post practice problems for the final on Tuesday; watch the webpage for details about the final review session
(4) The final is on Friday, 05/18 (2 weeks from today), 12:30-3:30pm, at 105 North Gate (school of journalism, immediately southeast of the Hearst&Euclid intersection. You may bring **3** 8.5×11 **pages of handwritten notes** — and no magnifying glass.

2. Warmup

Exercise 1. Fill in the blanks in Tables 1 and 2 (next page).

3. Chernoff bounds

Theorem 2 (Chernoff bound)**.** *If X is a sum of independent 0/1 (indicator) r.v.'s, and $\mathbb{E}[X] = \mu$, then* $\Pr[X > \alpha\mu] \leq e^{\alpha\mu - \mu - \alpha\mu \ln \alpha}$.

Exercise 3. A ping packet sent from my currently flaky DSL line to `inst.eecs` this morning arrives on average in 88 milliseconds, with a standard deviation of 83 milliseconds. Consider the following approaches to writing a script to automatically email my ISP and complain, and decide which bound would work best for each probability:

(1) Email whenever it sees a single ping with delay over 500 milliseconds. Bound the probability that a particular packet will cause a complaint.
(2) Email whenever it sees 90% of a sample of 100 packets take longer than 120 milliseconds. Bound the probability that a particular sequence of 100 packets would trigger this. What would you need to assume?
(3) Email whenever it sees 2 out of 3 consecutive packets take longer than 200 milliseconds. Why does this not get nicely bounded by the same technique as above?

In lecture, we derived Chernoff bounds by applying a Markov bound to $\Pr\left[\alpha^X > \alpha^{\alpha\mu}\right]$, and Luca mentioned that any monotonic, non-negative function would work as well.

Exercise 4. What happens if we decide to get an even stronger bound by applying Markov to $\Pr\left[2^{\alpha^X} > 2^{\alpha^{\alpha\mu}}\right]$?

4. Infinities

Exercise 5. Decide whether each of these is finite, countably infinite, or uncountable:

(1) The set of all bit strings ($\{\text{emptystring}, 0, 1, 00, 01, \ldots\}$).
(2) The set of all possible C programs.
(3) The set of all possible JPEG images.
(4) The set of all possible "general" (pixel) images, as an assignment of one of 16 million colors to each pair of integer coordinates

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(5) The set of all possible GPAs (assuming they're not rounded).
(6) The set of all possible Berkeley transcripts.
(7) The set of all possible ways to enumerate the sand grains at Stinson beach ("this one is grain #1, this one is grain #2, etc.").
(8) The set of all possible ways to assign an integer to all sand grains at Stinson beach.
(9) The set of all possible ways to assign "good" and "bad" to each possible bit string.

5. Challenge exercises

Exercise 6. Compare the cardinalities of the following to the cardinalities of $\mathbb{Z}$ and $\mathbb{R}$:

(1) $\mathbb{R}^2$ (points in the Euclidean plane)
(2) $\mathbb{R}^{\mathbb{Z}}$ (the number of functions from the integers to the reals; why does this notation make sense as an extention of the $\mathbb{R}^2$ notation?)
(3) $\mathbb{R}^{\mathbb{R}}$ (the number of functions from $\mathbb{R}$ to $\mathbb{R}$)
(4) $\mathbb{Z}^{\mathbb{R}}$ (the number of functions from the reals to the integers)
(5) $2^{\mathbb{R}^2}$ (first, figure out what this notation means)

	Need $\mathbb{E}[X]$?	Need $\mathrm{Var}[X]$?	Restrictions on X	Synopsis	Proof technique
Markov bound	Yes			Definite linear decrease (of $\Pr[X > k\mu]$, as k grows)	If it's very probable that it's much higher than mean, then $\Pr[\text{low}]$ can't be high enough to average out to the mean.
Chebyshev bound	Yes			Definite quadratic decrease (of $\Pr[\lvert X-\mu\rvert > k\sigma]$, as k grows)	
Chernoff bound	Yes			Definite exponential decrease (of $\Pr[X > k\mu]$, as k grows)	
Law of Large Numbers	Yes			**Asymptotically**, as n grows, X gets asymptotically close to the mean	If variance is finite, use Chebyshev; else, complicated
Central Limit Theorem	Yes			**Asymptotically**, as n grows, distribution of X looks like a normal of width $\sigma/\sqrt{n}$ around μ	Far beyond CS70 scope

Table 1. Summary of major probability theorems from CS70

	Pros	Cons
Markov bound		
Chebyshev bound		
Chernoff bound		
Law of Large Numbers		
Central Limit Theorem		

Table 2. Comparison of probability theorems