CS 70 SPRING 2007 — DISCUSSION #15 "ANSWER SHEET"

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	Need $\mathbb{E}[X]$?	Need $\text{Var}[X]$?	Restrictions on X	Synopsis	Proof technique
Markov bound	Yes	No	Non-negative	Definite linear decrease (of $\Pr[X > k\mu]$, as k grows)	If it's very probable that it's much higher than mean, then $\Pr[\text{low}]$ can't be high enough to average out to the mean.
Chebyshev bound	Yes	Yes	NONE	Definite quadratic decrease (of $\Pr[\|X-\mu\| > k\sigma]$, as k grows)	Apply Markov to $Y = (X-\mu)^2$
Chernoff bound	Yes	No	X must be a sum of independent indicators	Definite exponential decrease (of $\Pr[X > k\mu]$, as k grows)	Apply Markov to $Y = \alpha^X$
Law of Large Numbers	Yes	No	X is the average of n i.i.d. variables	**Asymptotically**, as n grows, X gets asymptotically close to the mean	If variance is finite, use Chebyshev; else, more complicated
Central Limit Theorem	Yes	Yes	X is the average of n i.i.d. variables, with finite expectation and variance	**Asymptotically**, as n grows, distribution of X looks like a normal of width $\sigma/\sqrt{n}$ around μ	Far beyond CS70 scope

	Pros	Cons
Markov bound	Applies to all non-negative variables Just need to know $\mathbb{E}[X]$	Very weak bound
Chebyshev bound	Applies to all variables!	Major: Weak bound Minor: Need to know variance
Chernoff bound	Very strong bound	Sum-of-indicators is a very narrow species of random variable
Law of Large Numbers	(almost just a special case of central limit theorem, but doesn't require finite variance)	Only asymptotic — "eventually, you'll almost definitely be very close to the mean" Only applies to average of independent samples from the same r.v.
Central Limit Theorem	Very complete description of the shape of the distribution when n grows asymptotically	On any finite example, you have to *guess* that n is big enough to make the normal a good *approximation*. In this class, you can't make any definitive statements based on the CLT. Only applies to average of independent samples from the same r.v.