CS 70 SPRING 2007 — DISCUSSION #14

ALEX FABRIKANT

1. Administrivia

(1) There is an extra, optional "supplementary" section on Wednesdays, 4:30-6:30pm, in 320 Soda.
(2) Homework 11 will be posted tonight; it'll be due on Wednesday, 05/02

2. Warmup

Pick 3 real numbers, r_1, r_2, and r_3 uniformly at random. When asked to compute the probability that r_3 is the biggest of them all:

(1) Luca says, "The probability that $r_3 > r_2$ is $\frac{1}{2}$ (given r_2, r_3 is equally likely to be on either side of it), and the probability that $r_3 > r_1$ is $\frac{1}{2}$ (same argument). The probability that both happen is $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$.
(2) Alex says, "Pick r_1 and r_2. There's only a finite amount of space between r_1 and r_2, but an infinite amount of space to either side of them. Thus the probability that r_3 is between them is infinitely small, and since it's equally likely to be either greater than both or smaller than both, the probability that it's the greatest is $\frac{1}{2}$.
(3) Vahab says, "Each permutation of the three numbers is equally likely, and r_3 comes out the biggest in 2 out of 6 cases, so the probability is $\frac{1}{3}$.

Exercise 1. What's going on here?

3. Continuous random variables

Exercise 2. What happens to the probability density function of the distribution which is exactly 10 with probability 1/2, and otherwise is uniformly distributed on the interval $[0, 5]$?

Exercise 3. Let X be a random variable with probability density function $f(x) = \frac{1}{(1+x)^2}$. Compute:

(1) $\Pr[X > 3]$
(2) Find $\mathbb{E}[X]$.

4. Central Limit Theorem

Exercise 4. You roll 420 standard dice. Use the approximation given by the Central Limit Theorem to answer the following questions. Can we be sure that that their sum will be less than 1500 at a 84.4% confidence level? In which interval should the sum fall at a 98% confidence level?

Exercise 5. Compare the bound given by Chebyshev Inequality and that given by the Central Limit Theorem for a variable $X = X_1 + X_2 + X_3 + \ldots + X_n$, where the $\{X_i\}$'s are i.i.d with mean μ and variance σ and n is large. How much better is the CLT bound for a 1σ variation? For a 2σ variation?

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