CS 70 SPRING 2007 — DISCUSSION #12

ALEX FABRIKANT

1. Administrivia

(1) Midterm 2 stats: mean 31.1, median 31 (this section: 34!), stdev 9.87
(2) Homework 9 has been posted; it's due on Tuesday, 4/17

2. Warm-up

Exercise 1. Let X be a random variable equal to 1 with probability 0.5, and to -1 with probability 0.5. Compute: $E[X^2]$, $E[X]^2$, $\text{Var}[X]$.

3. Holy socks

In my sock drawer, there's some number of socks. Some of the socks have holes. Some have multiple holes. Suppose I pull out a sock at random. Let X be the number of holes this sock has.

Exercise 2. What is the sample space here (what are the outcomes)? What is the range of the random variable X?

I put that sock back, and pull out a sock at random again. Let Y be the number of holes this sock has.

Exercise 3. Why is Y independent of X?

Exercise 4. Show that, since Y and X are independent, we know that $\text{Var}[X + Y] = \text{Var}[X] + \text{Var}[Y]$ despite not knowing anything about the actual distributions of X or Y

Let $Z = X$.

Exercise 5. What is the difference between Y and Z? Is Z independent of X?

Exercise 6. Compute $\text{Var}[X + Z]$.

4. Binomial Distribution

$$X \sim \text{Bin}(n, p) : \Pr(X = k) = \binom{n}{k} p^k (1 - p)^{n-k}$$

Exercise 7. If there are 10 socks in the drawer, and 4 of them have no holes, what's the probability that after pulling out two random socks at the same time 5 times (and putting back each pair after you look at it), you pull out a pair with no holes 4 out of 5 times.

5. Poisson Distribution

$$X \sim \text{Poi}(\lambda) : \Pr(X = k) = \frac{\lambda^k e^{-\lambda}}{k!}$$

Exercise 8. The probability of the Perimeter shuttle appearing during any given minute outside Cory is 20%. What's the probability that no shuttles will appear within a particular span of 5 minutes, assuming the shuttles operate independently?

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6. Geometric Distribution

$$X \sim \text{Geo}(p) : \Pr(X = k) = (1-p)^{k-1}p$$

Exercise 9. James Bond is imprisoned in a cell from which there are three possible ways to escape: an air-conditioning duct, a sewer pipe and the door (which is unlocked). The air-conditioning duct leads him on a two-hour trip whereupon he falls through a trap door onto his head, much to the amusement of his captors. The sewer pipe is similar but takes five hours to traverse. Each fall produces temporary amnesia and he is returned to the cell immediately after each fall. Assume that he always immediately chooses one of the three exits from the cell with probability $\frac{1}{3}$.

(1) What is the probability of escaping after seven attempts?
(2) On the average, how long does it take before he realizes that the door is unlocked and escapes?

7. A different kind of "challenge exercise"

Exercise 10. Figure out what specific parts of the material on combinatorics and/or probability were the most confusing to you and your friends in the class thus far, and tell me what you come up with. I'm considering running something like an extra discussion on a weekly basis to help you guys get a better feel for the topics recently presented, and any input would be useful.