CS 70 SPRING 2007 — DISCUSSION #1

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1. Administrivia

(1) Course Information
- The first homework is **due January 30th** in 283 Soda Hall, and will be made available at `http://inst.eecs.berkeley.edu/~cs70/` on Tuesday, January 23rd
- You are encouraged to work on the homework in groups of 3-4, but write up your submission *on your own*. Cite any external sources you use.

(2) Discussion Information
- If you have a clash, it is OK to attend a section different to your enrolled/wait-listed one. Just be sure to show up so that we can 'assign' you somewhere based on the rolls taken in sections in the first few weeks.
- Section notes like these will be posted on the course website.
- Feel free to contact the GSI's via e-mail, or the class staff and students through the newsgroup, ucb.class.cs70, if you have a question.

2. A warm-up exercise

Exercise 1. Write down the truth table for $\neg A \to B$. What else is this operation on A and B known as?

3. Knights and Knaves: Fun with Propositional Logic

Knights are always truthful while knaves are consistent liars. With this in mind, consider the following conversation between Alice and Bob.

Alice: "At least one of us is a knight."
Bob: "At least one of us is a knight."

How might we determine whether Alice is a knight or a knave? As we're dealing with the truth of statements (i.e. that "Alice is a knight" and the same for Bob), propositional logic should come to mind! Let's begin by labeling the two propositions we're interested in:

$$P = \text{“Alice is a knight”}$$
$$Q = \text{“Bob is a knight”}$$

Exercise 2. Using the propositional machinery discussed in class, determine exactly what can be deduced from the above facts/statements.

(i) As a first step, write out the *four* distinct statements in terms of P and Q that must be true. (**hint:** what if Alice is/is not a knight, what about Bob?).

(ii) P must be either true or false, as must be Q. Also your four logical propositions must be true. Using a truth table determine which truth assignments to P and Q are consistent with the truth of your four statements. From this deduce what can be said about the truth of P and Q.

□

Using the same steps we can solve many variants of the knights and knaves problem – this is *generalization* in problem solving.

Exercise 3. Repeat the previous exercise after Alice says "At least one of us is a knight" and Bob says "We're both of the same (knight/knave) type."

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4. Quantifier Practice

Consider the false statement "For each x in $\mathbb{R}$, $x^2 \geq x$" (consider $0 < x < 1$). What is the negation of this statement? Is it "For each x in $\mathbb{R}$, $x^2 < x$"? No, because this statement is still false (e.g. consider $x > 1$). So what is going wrong here?

Let $P(x)$ be the proposition "$x^2 \geq x$" with x taken from the universe of real numbers $\mathbb{R}$. Then our original statement is succinctly written as $\forall x, P(x)$. Using DeMorgan's laws, we get $\neg\forall x, P(x) \equiv \exists x, \neg P(x)$ or "There exists a real x for which $x^2 < x$."

We can chain together quantifiers in any manner we please: $\forall x, \exists y, \forall z, P(x, y, z)$ and negate it using the same rules discussed above. By applying the rules in sequence, we get that

$$\neg(\forall x.\exists y.\forall z.P(x, y, z))$$
$$\exists x.\neg(\exists y.\forall z.P(x, y, z))$$
$$\exists x.\forall y.\neg(\forall z.P(x, y, z))$$
$$\exists x.\forall y.\exists z, \neg P(x, y, z)$$

The $\neg$ "bubbles down", flipping quantifiers as it goes. The following problem comes from Question 14 in the Mathematics Subject GRE Sample Test:

Exercise 4. Let $\mathbb{R}$ be the set of real numbers and let f and g be functions from $\mathbb{R}$ to $\mathbb{R}$. The negation of the statement

"For each s in $\mathbb{R}$, there exists an r in $\mathbb{R}$ such that if $f(r) > 0$, then $g(s) > 0$."

is which of the following?

(A) For each s in $\mathbb{R}$, there exists an r in $\mathbb{R}$ such that $f(r) \leq 0$ and $g(s) > 0$.
(B) There exists an s in $\mathbb{R}$ such that for each r in $\mathbb{R}$, $f(r) \leq 0$ and $g(s) \leq 0$.
(C) There exists an s in $\mathbb{R}$ such that for each r in $\mathbb{R}$, $f(r) \leq 0$ and $g(s) > 0$.
(D) There exists an s in $\mathbb{R}$ such that for each r in $\mathbb{R}$, $f(r) > 0$ and $g(s) \leq 0$.
(E) For each s in $\mathbb{R}$, there exists an r in $\mathbb{R}$ such that $f(r) \leq 0$ and $g(s) \leq 0$.

Use the tools covered above. (**hint:** what happens when you negate an implication? Try rewriting the statements in propositional logic, e.g. replacing $f(r) > 0$ with $P(r)$ and $g(s) > 0$ with $Q(s)$). □

Exercise 5. Express the following statements using propositional logic:

(1) "There exist at least 2 distinct integers x that satisfy $P(x)$."
(2) "There exist at most 2 distinct integers x that satisfy $P(x)$."
(3) "There exist exactly 2 distinct integers x that satisfy $P(x)$."

5. Challenge problems

Most weeks, we'll try to throw in an extra problem or two for those of you who want an extra challenge. We'll usually not get to these in section — they're for you to think about on your own time, but we welcome questions about them after section or in office hours.

Exercise 6. Time for more cup-flipping! The setup: There are two rooms with a solid wall between them. One room contains a one-legged cross-shaped table which rotates around its center. It's designed so that whenever it's spun, it rotates for a while but always stops with the four points of the cross aligned along compass directions (i.e. it only rotates by multiples of 90°). The table has cups at each point of the cross, and an operator who takes instructions from you. You're in the other room, and you don't know what positions the cups start out in — each can be right side up or upside down. The only thing you're allowed to do is give the operator instructions to flip some of the cups (eg "flip the south and east cups", which counts as one operation). The operator obeys these, but after every such operation, he spins the table.

Your goal is to make all the cups face in the same direction. The operator only speaks up once you've achieved the goal to tell you you're done. Is there a way for you to make sure you reach the goal eventually? If so, what is the smallest number of operations you need to make the operator do before the goal is definitely achieved?